

This task was developed by high school and postsecondary mathematics and agriculture sciences educators, and validated by content experts in the Common Core State Standards in mathematics and the National Career Clusters Knowledge & Skills Statements. It was developed with the purpose of demonstrating how the Common Core and CTE Knowledge & Skills Statements can be integrated into classroom learning – and to provide classroom teachers with a truly authentic task for either mathematics or CTE courses.

TASK: GRAIN STORAGE

TARGET COMMON CORE STATE STANDARD(S) IN MATHEMATICS:

G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

7.RP.3 Use proportional relationships to solve multi-step ratio and percent problems.

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

TARGET STANDARDS FOR MATHEMATICAL PRACTICES

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.4 Model with mathematics.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

TARGET CAREER AND TECHNICAL EDUCATION (CTE) KNOWLEDGE & SKILLS STATEMENTS:

AGPB01.04.01 Store crops using methods that apply fundamentals of plant management.

AGPG01.04 Assess and manage inventory using Agriculture, Food, and Natural Resources (AFNR) industry concepts and inventory control practices to ensure adequate inventory for business demand.

RECOMMENDED COURSE:

Geometry; Integrated Math I or II; Agribusiness I

ADDITIONAL INSTRUCTIONS:

This task might be completed in one class period with extra time allowed for the design portion.

* Modeling standards appear throughout the CCSS high school standards and are indicated by a star symbol (*).

About the Common Core State Standards in Mathematics

The Common Core State Standards (CCSS) for Mathematics are organized by grade level in grades K–8. At the high school level, the standards are organized by conceptual category (number and quantity, algebra, functions, geometry, and probability and statistics), showing the body of knowledge students should learn in each category to be college and career ready, and to be prepared to study more advanced mathematics. The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. www.corestandards.org

About the Common Core State Standards in English Language Arts/Literacy

The Common Core State Standards (CCSS) for ELA/Literacy are organized by grade level in grades K–8. At the high school level, the standards are organized by 9-10 and 11-12 grade bands. Across K-12 there are four major strands: Reading, Writing, Speaking and Listening, and Language. The CCSS also include Standards for Literacy in History/Social Studies, Science, and Technical Subjects, with content-specific (Reading and Writing) literacy standards provided for grades 6-8, 9-10, and 11-12, to demonstrate that literacy needs to be taught and nurtured across all subjects. www.corestandards.org

About the Career Cluster Knowledge and Skill Statements

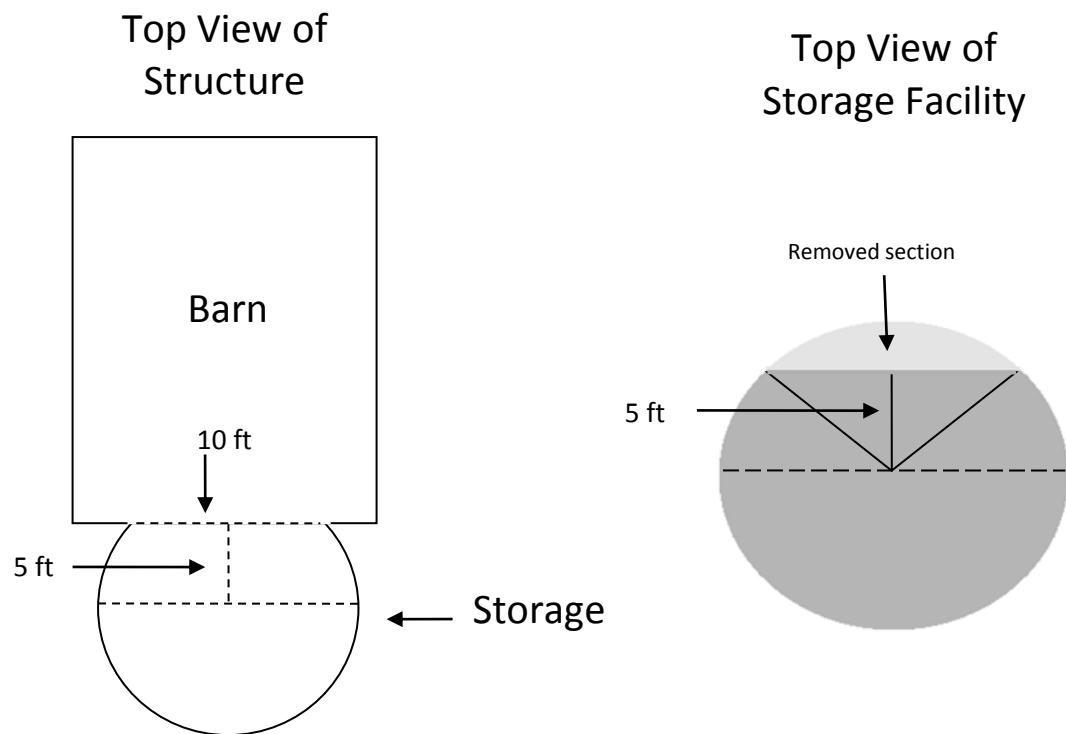
As an organizing tool for curriculum design and instruction, Career Clusters™ provide the essential knowledge and skills for the 16 Career Clusters™ and their Career Pathways. It also functions as a useful guide in developing programs of study bridging secondary and postsecondary curriculum and for creating individual student plans of study for a complete range of career options. As such, it helps students discover their interests and their passions, and empowers them to choose the educational pathway that can lead to success in high school, college and career. <http://www.careertech.org/career-clusters/resources/clusters/agriculture.html>. Although not included in this template, all Clusters and Pathways have Foundational Academic Expectations and Essential Knowledge & Skills Statements, which, in some cases, overlap with the Common Core State Standards.

KEY TERMS

- Bushels
- Isosceles Right Triangles
- Bisected
- Segment and Sector of a circle
- Chord, Radii, Central angle

GRAIN STORAGE – *The Task*

John is a producer who stores wheat in a storage facility located on his farm. The cylinder-shaped storage facility has one flat, rectangular face that rests against the side of his barn. The barn is 5 feet from the center of the storage facility. The height of the stored grain is 30 feet (when full to capacity) and the face resting against the barn is 10 feet wide. This storage facility holds the exact amount of wheat he uses per year.



1. Determine the capacity of John's storage in cubic feet.
2. If one bushel is 1.25 cubic feet, how many bushels can be stored?
3. Since John will be acquiring more acres, he foresees an increase in production of 23% for the next year. What would be the total amount of storage required in cubic feet and bushels for next year?
4. John's current facility is outdated. Design a new storage facility. Give dimensions and provide a diagram. Consider vertical, horizontal, temporary, and/or mobile facilities. Justify your final choice of design. *Show your work!*

GRAIN STORAGE – Possible Solution(s)

1. The intersection of the barn and the storage facility allows the construction of two right triangles, as seen from the top view of the storage facility. The distance between the side of the barn and the center of the storage facility is given as 5 ft. Using the properties of triangles and the fact that if the radius of a circle is perpendicular to a chord, then it bisects it, we can deduce that the two triangles are isosceles right triangles, with legs 5 ft. in length.

We can use the Pythagorean Theorem, or the properties of special right triangles, to find the length of the hypotenuse of one of the triangles, which is also the radius of the circle, to be $5\sqrt{2}$ ft.

The area of the circle, as seen from the top view of the storage facility, would be found using the formula:

$$A = \pi r^2$$

$$A = \pi(5\sqrt{2})^2 = 50\pi \text{ ft}^2, \text{ or approximately } 157 \text{ ft}^2.$$

The larger triangle formed by the union of the two isosceles right triangles has a base of 10 and a height of 5, as given in the task. The area of this triangle can be found using the formula:

$$A = \frac{1}{2} bh$$

$$A = \frac{1}{2} (10)(5) = 25 \text{ ft}^2$$

Since the two small isosceles right triangles “meet at the center of the circle” with their combined 45 degree angles, the sector of the circle that encompasses the triangles has a 90 degree central angle and is a quarter of the circle. The remaining sector is three-fourths of the circle. The area of the three-fourths of the circle plus the area of the triangle formed by the two isosceles right triangles equals the area of the base of the storage facility.

$$\text{Area of the base of the storage facility} = \frac{3}{4} (50\pi) \text{ ft}^2 + 25 \text{ ft}^2 = 37.5\pi \text{ ft}^2 + 25 \text{ ft}^2 = \text{approximately } 142.8 \text{ ft}^2.$$

To find the capacity of the storage facility, in cubic feet, the student must then multiply the area of the base by 30 ft., the height of the grain when the storage facility is filled to capacity.

$$V = (142.8 \text{ ft}^2) (30 \text{ ft}) = 4284 \text{ ft}^3. \text{ Therefore, the capacity of the storage facility is approximately } 4284 \text{ ft}^3.$$

2. Using unit analysis to change cubic feet to bushels:

$$\frac{1 \text{ bushel}}{1.25 \text{ ft}^3} \cdot 4284 \text{ ft}^3 = \frac{4284}{1.25} \text{ bushels} = \underline{3427.2 \text{ bushels}}$$

3. To find the total amount of storage required in cubic feet and bushels for next year, we first need to find 23% of the current capacity since a 23% increase in production is projected.

$$0.23 \times 4284 \text{ ft}^3 = \text{approximately } 985.3 \text{ ft}^3$$

If this is added to the original capacity, we find next year's projected storage needs to be, in cubic feet:

$$4284 \text{ ft}^3 + 985.3 \text{ ft}^3 = \underline{5269.3 \text{ ft}^3} \text{ or } 5269.3 / 1.25 = \underline{4215.4 \text{ bushels}}$$

An alternative solution strategy would be to multiply the capacity of the storage facility by 1.23, arriving at the same solution.

4. This is an open-ended design question, and students may approach it in a variety of ways.

For example, a student may decide that they would want to keep the base of the storage facility the same shape and size and simply increase the height to accommodate additional grain in future years.

Other options could involve changing the area of the base of the storage facility and possibly also the height, as needed. Another design option might be to build the storage facility so that it does not rest against the side of the barn, as it now does. Regardless of the design selected by the student, they must provide a diagram, dimensions, and justify their decisions with respect to the size, orientation, and placement of the storage facility.

GRAIN STORAGE – *Possible Extensions*

The extensions below represent potential ways in which mathematics and/or CTE teachers can build on the task above. All of the extensions are optional and can be used in the classroom, as homework assignments, and/or as long-term interdisciplinary projects.

1. Is it more profitable to build a new facility or consider a temporary solution? Using current market prices and local regulations, present an argument for why your solution is a good one.
2. Students divide into groups and consider various features of the new storage solutions proposed by other classmates. For example, what is the total area of the galvanized metal sheets required (before bent) given the size of the facility? For discussion: What are the differences in gauges of tin? What is the value of corrugation?
3. Compare the storage of corn vs. wheat with respect to weight and volume.
4. Discuss and compare gross weight vs. net weight of a tanker truck.
5. Examine local regulations and major laws that might impact on the construction of the new storage facility. Take these into account in the design of the new facility.
6. Collect research and gather public input to inform design of the new storage facility. Use this input to come up with alternative designs, and evaluate the various designs based on issues and regulations impacting AFNR management.
7. In designing the new storage facility, analyze various design options for safety and environmental issues. Present design options that address these issues, and offer a design recommendation.
8. John wants to avoid having another situation in the near future where grain production rises so steeply in a one-year period that he is unprepared. Describe inventory control measures that he could put in place to monitor inventory levels. Also discuss steps that he could put in place to monitor current grain production and predict future production.

GRAIN STORAGE – *Appendix: Alignment Ratings*

The rating system used in the following charts is as follows:

3 EXCELLENT ALIGNMENT:

The content/performance of the task is clearly consistent with the content/performance of the Common Core State Standard.

2 GOOD ALIGNMENT:

The task is consistent with important elements of the content/performance of the CCSS statement, but part of the CCSS is not addressed.

1 WEAK ALIGNMENT:

There is a partial alignment between the task and the CCSS, however important elements of the CCSS are not addressed in the task.

N/A:

For Mathematical Practices a content rating does not apply.

In the charts **C = Content Rating** and **P = Performance Rating**

COLOR KEY

- **Black** = Part of CCSS/K&S Statement aligned to task
- **Gray** = Part of CCSS/K&S Statement *not* aligned to task

Task-to-Mathematical Practice Alignment Recording Sheet

Task Name	Aligned CCSS Mathematical Practice Standards	C	P	Alignment Comments (Standards selection, partial alignments, reasons for rating, etc)	Task Comments (Strengths, weaknesses, possible improvements, effectiveness, etc)
GRAIN STORAGE	MP.1 Make sense of problems and persevere in solving them.	N/A	3	This task requires that students analyze givens, constraints, relationships, and goals. They must make conjectures about the form and meaning of the solution and plan a solution pathway. They must make sure their answers make sense in the context of the problem.	This is a multi-stage problem with real life applications and considerations. This task asks students to apply the mathematics they know to solve a workplace problem. Students must identify measurements and quantities and use perform quantitative calculations.
	MP.2 Reason abstractly and quantitatively.	N/A	3	This task requires that students make sense of quantities and their relationships in problem situations. They must create a coherent representation of the problem at hand, considering units and attending to the meaning of quantities.	
	MP.4 Model with mathematics.	N/A	3	This task asks students to apply the mathematics they know to solve a workplace problem. They use measurement and geometric knowledge and skills and interpret their results in the context of the problem.	
	MP.6 Attend to precision.	N/A	3	This task requires accurate calculations and solutions that are expressed with a degree of precision appropriate for the problem and the units involved.	
	MP.7 Look for and make use of structure.	N/A	3	This task requires that students use geometric structure and step back for an overview and shift of perspective. They must recognize the significance of the 5 ft. line segment, and the resulting two right triangles it can create in order to help solve the problem.	

Task-to-Common Core State Standards Alignment Recording Sheet

Task Name	Aligned CCSS Content Standards	C	P	Alignment Comments (Standards selection, partial alignments, reasons for rating, etc.)	Task Comments (Strengths, weaknesses, possible improvements, effectiveness, etc.)
GRAIN STORAGE	G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	2	2	This task uses central angle and the chord / radii relationship. As students explain or show their work the relationships would be indirectly identified but not necessarily described.	This complex real-world task involving multiple calculations that requires students to apply geometric concepts in a concrete way.
	G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*	3	3	In question 4 the student must apply geometric methods to design a new storage facility to update the current one.	
	G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	2	3	The students must use the Pythagorean Theorem, either directly or indirectly (special triangle relationships) but they do not need to use trigonometric ratios.	
	8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	3	3	The student must use the Pythagorean Theorem, either directly or indirectly (special triangle relationships) to determine the length of the hypotenuse (the radius of the circle) in the created right triangles.	
	7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. <i>For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.</i>	3	2	Students must use unit rates, such as cubic feet per bushel, but do not have to compute the rate, as it is given in the problem.	
	7.RP.3 Use proportional relationships to solve multi-step ratio and percent problems.	3	3	In question 3 the student must compute the storage capacity needed after a 23% increase in production.	
	7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	2	2	Circumference is not referenced or needed in this task.	

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	2	3	This task uses only circles, triangles, and cylinders.	
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* Modeling standards appear throughout the CCSS high school standards and are indicated by a star symbol (*).

Task-to-National Career Cluster Knowledge & Skills Statements Alignment Recording Sheet

Task Name	Aligned National Career Cluster Knowledge & Skills Statements	C	P	Alignment Comments	Task Comments
Grain Storage	AGPB01.04.01 Store crops using methods that apply fundamentals of plant management.	2	1	The context of storing crops in the problem allows for determining one aspect of crop storage—capacity. The task does fully explore the fundamentals of crop storage.	The task provides an opportunity to evaluate current and future needs associated with onsite inventory storage of grain.
	AGPG01.04 Assess and manage inventory using AFNR industry concepts and inventory control practices to ensure adequate inventory for business demand.	2	3	The task provides an opportunity for students to explore existing inventory control methods and develop strategies for future expectations based upon a changing demand of growth.	