This task was developed by high school and postsecondary mathematics and health sciences educators, and validated by content experts in the Common Core State Standards in mathematics and the National Career Clusters Knowledge & Skills Statements. It was developed with the purpose of demonstrating how the Common Core and CTE Knowledge & Skills Statements can be integrated into classroom learning – and to provide classroom teachers with a truly authentic task for either mathematics or CTE courses.

TASK: RANGE OF MOTION

TARGET COMMON CORE STATE STANDARD(S) IN MATHEMATICS:

G-SRT.8 Use trigonometric ratios and the Pythagorean theorem to solve right triangles in applied problems.

N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*

7.RP3: Use proportional relationships to solve multistep ratio and percent problems.

6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.

TARGET STANDARDS FOR MATHEMATICAL PRACTICES

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.6 Attend to precision.

TARGET COMMON CORE STATE STANDARD(S) IN ELA/LITERACY:

RST.9-10.3 Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.

RST.9-10.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 9–10 texts and topics.*

TARGET CAREER AND TECHNICAL EDUCATION (CTE) KNOWLEDGE & SKILLS STATEMENTS:

HLC01.01.01: Use knowledge of human structure and function to conduct health care role.

HLPB02.01: Utilize the processes to assess and report patient's/client's health status in order to monitor and document patient progress.

HLPA06.01: Evaluate patient/client needs, strengths, and problems in order to determine if treatment goals are being reached.

HLPB06.01: Interpret any given procedure, demonstrate knowledge of the purpose for each procedure, and perform the specific procedure to create diagnostic results.

RECOMMENDED COURSE(S):

Geometry, Integrated Math II

ADDITIONAL INSTRUCTIONS:

This task can be completed in one class period and might be used as a formative or summative assessment.

* Modeling standards appear throughout the CCSS high school standards and are indicated by a star symbol (*).

About the Common Core State Standards in Mathematics
The Common Core State Standards (CCSS) for Mathematics are organized by grade level in grades K–8. At the high school level, the standards are organized by conceptual category (number and quantity, algebra, functions, geometry, and probability and statistics), showing the body of knowledge students should learn in each category to be college and career ready, and to be prepared to study more advanced mathematics. The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. www.corestandards.org

About the Common Core State Standards in English Language Arts/Literacy
The Common Core State Standards (CCSS) for ELA/Literacy are organized by grade level in grades K–8. At the high school level, the standards are organized by 9-10 and 11-12 grade bands. Across K-12 there are four major strands: Reading, Writing, Speaking and Listening, and Language. The CCSS also include Standards for Literacy in History/Social Studies, Science, and Technical Subjects, with content-specific (Reading and Writing) literacy standards provided for grades 6-8, 9-10, and 11-12, to demonstrate that literacy needs to be taught and nurtured across all subjects. www.corestandards.org

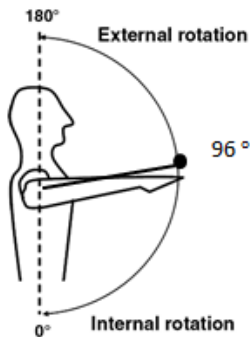### *About the Career Cluster Knowledge and Skill Statements*
As an organizing tool for curriculum design and instruction, Career Clusters™ provide the essential knowledge and skills for the 16 Career Clusters™ and their Career Pathways. It also functions as a useful guide in developing programs of study bridging secondary and postsecondary curriculum and for creating individual student plans of study for a complete range of career options. As such, it helps students discover their interests and their passions, and empowers them to choose the educational pathway that can lead to success in high school, college and career. http://www.careertech.org/career-clusters/resources/clusters/health.html. Although not included in this template, all Clusters and Pathways have Foundational Academic Expectations and Essential Knowledge & Skills Statements, which, in some cases, overlap with the Common Core State Standards.
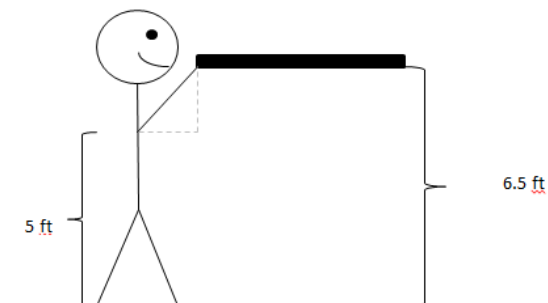

KEY TERMS

- Active Range of Motion (AROM)
- Goniometer
- Internal / external rotation
- Angle of motion, angle of elevation
- Insurance goals for physical therapy
- Occupational goals for physical therapy

RANGE OF MOTION – *The Task*

Joe was cycling when he lost control of his bike, causing him to fall and dislocate his shoulder. His doctor has prescribed 6 weeks of physical therapy. The insurance goal of his therapy is to increase his active range of motion (AROM) in the upward direction (external rotation) by an additional 3% each week, starting from his initial reading on the goniometer[1] of an AROM of 96° (based on the radius of a circle and as depicted in the following diagram).

1. Assuming Joe meets his insurance goal, create a table tracking Joe's projected progress over the six weeks of physical therapy, and graph each location on the circle, representing the range of motion.

2. At the end of six weeks, how many degrees of motion, in the upward direction, will have been gained?

3. As part of his occupational goal, Joe is working toward being able to put away dishes on a shelf that is 6.5 ft. from the floor. Assuming Joe's shoulder height is 5 ft. from the floor and his arm is 30 in. long, what angle of motion does he need to have to reach the shelf and achieve his goal?
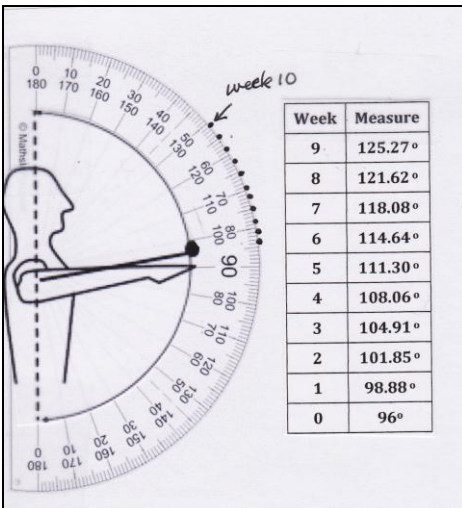

4. How many weeks of physical therapy will it take Joe to reach the 6.5 ft. shelf?

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[1] Teachers may show this short video to explain what a goniometer is and how it is used to measure range of motion
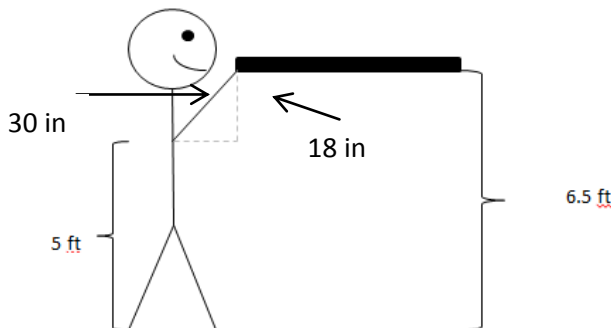www.cdc.gov/NCBDDD/video/JointROM/Goniometer/

RANGE OF MOTION – *Possible Solutions*

1. The student can either multiply each measure by 3% (= 0.03) and add the measure, or multiply each measure by 1.03. Using a spreadsheet or programmable calculator will greatly expedite the data tabulation process.

Week	Measure
9	125.27°
8	121.62°
7	118.08°
6	114.64°
5	111.30°
4	108.06°
3	104.91°
2	101.85°
1	98.88°
0	96°

Week	AROM
0	**96°**
1	96(1.03) = 98.88°
2	98.88(1.03) = 101.85°
3	101.85(1.03) = 104.91°
4	104.91(1.03) = 108.06°
5	108.06(1.03) = 111.30°
6	111.30(1.03) = 114.64°

2. Six-week range – initial range = 114.64 – 96 = **18.64°** (Note: Students may round answer this to 19°.)

3. Use the diagram below to find the angle of motion:

30 in

18 in

6.5 ft

5 ft

We can determine the measurements of two sides of the right triangle with Joe's arm as the hypotenuse, as shown above. We know Joe's arm is 30 inches long, and can determine the length of the vertical leg of the triangle by subtracting the height of Joe's shoulder from the height of the shelf. We see that the height of the triangle is 6 ½' – 5' = 1 ½ ft or **18 inches** (or 78 in – 60 in = 18 in).

Now to find the angle of elevation for Joe's arm when he reaches to the shelf we use the right triangle. Since it is a right triangle, and the two known triangle side lengths are the side opposite the angle and the hypotenuse, we can use trigonometry to find the angle.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{18''}{30''} = 0.6$$

If sin θ = 0.6, then **θ = 36.87°**

Since the triangle drawn is starting at the 90° AROM, this angle must be added to 90°. Therefore, the full angle of motion required to reach the shelf is
90° + 36.87° = 126.87°

4. The student can continue the chart from question 1.

Week	AROM
7	114.64(1.03) = 118.08 °
8	118.08(1.03) = 121.62 °
9	121.62(1.03) = 125.27 °
10	125.27(1.03) = 129.03 °
11	129.03(1.03) = 132.90 °

We see at the end of week 10 that 129.03 ° > 126.87 °.
Therefore, Joe will be able to reach his goal during the **10th week.**

RANGE OF MOTION - *Possible Extensions*

The extensions below represent potential ways in which mathematics and/or CTE teachers can build on the task above. All of the extensions are optional and can be used in the classroom, as homework assignments, and/or as long-term interdisciplinary projects.

1. Using a goniometer, have students work with a partner to measure each other's range of motion required to brush their hair.

2. Then have students, using string, tie their wrists to their (a) thigh (b) waist, etc., and have them measure each other's ROM with a goniometer. Start with 36 inches of string and then add 12 inches (48 inches total) and take away 12 inches (24 inches total) and measure the adjusted ROMs. Note the limitations in activities of daily living that are restricted by limited ROM.

3. Create documentation monitoring Joe's progress over six weeks – and his ROM at the end of the six weeks – including recommendations for next steps at the end of his prescribed physical therapy.

4. Document Joe's initial AROM using the range of joint motion evaluation chart.
   http://www.dshs.wa.gov/pdf/ms/forms/13_585a.pdf

The rating system used in the following charts is as follows:

3   **EXCELLENT ALIGNMENT:**
The content/performance of the task is clearly consistent with the content/performance of the Common Core State Standard.

2   **GOOD ALIGNMENT:**
The task is consistent with important elements of the content/performance of the CCSS statement, but part of the CCSS is not addressed.

1   **WEAK ALIGNMENT:**
There is a partial alignment between the task and the CCSS, however important elements of the CCSS are not addressed in the task.

N/A:
For Mathematical Practices a content rating does not apply.

In the charts **C = Content Rating** and **P = Performance Rating**

COLOR KEY
- **Black** = Part of CCSS/K&S Statement aligned to task
- **Gray** = Part of CCSS/K&S Statement *not* aligned to task

Task-to-Mathematical Practice Alignment Recording Sheet

Task Name	Aligned CCSS Mathematical Practice Standards	C	P	Alignment Comments (Standards selection, partial alignments, reasons for rating, etc)	Task Comments (Strengths, weaknesses, possible improvements, effectiveness, etc)
RANGE OF MOTION	**MP.1:** Make sense of problems and persevere in solving them.	N/A	3	For this task, students analyze the given information, the constraints, relationships, and goals of the task. They must make conjectures about the form and meaning of the solution and plan a solution pathway. They must check the reasonableness of their solution, continually asking themselves, "Does this make sense?" While the task requires routine math, the student must persevere in the calculations and analysis.	This is a multi-stage problem with real life applications and considerations. Students must identify quantities from practical situations, and use accurate quantitative calculations.
	MP.2: Reason abstractly and quantitatively.	N/A	3	This task involves quantitative relationships. It requires that students make sense of quantities and their relationships in the problem situation. They must attend to the meaning of the quantities and pay attention to units as they represent the quantities in their solution. They must de-contextualize to solve the problem and then re-contextualize to interpret the results.	
	MP.6 Attend to precision.	N/A	3	Students need to calculate accurately and express answers with a degree of precision appropriate to the problem. Students are also asked to graph locations representing range of motion on a circle, which calls for a level of precision.	

Task-to-Common Core State Standards Alignment Recording Sheet

Task Name	Aligned CCSS Content Standards	C	P	Alignment Comments (Standards selection, partial alignments, reasons for rating, etc.)	Task Comments (Strengths, weaknesses, possible improvements, effectiveness)
RANGE OF MOTION	**G-SRT.8:** Use trigonometric ratios and the Pythagorean theorem to solve right triangles in applied problems.	2	3	Trigonometric ratios are essential in Question 3. However there is no requirement to use the Pythagorean relationship in solving for side lengths.	This real-world task combines calculation with decimals, percentages, and trigonometric ratios.
	N-Q.3: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*	3	3	The student must make precision decisions when reporting results of angle measures.	
	7.RP3: Use proportional relationships to solve multistep ratio and percent problems.	2	3	Percentages are used throughout the task in the increase of range of motion. This task does not technically require the use of ratios.	
	6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.	3	3	For this task students must understand that 3% is the constant rate of change for the range of motion and use that to find resulting AROM's at different weeks.	

* Modeling standards appear throughout the CCSS high school standards and are indicated by a star symbol (*).

Task-to-National Career Cluster Knowledge & Skills Statements Alignment Recording Sheet

Task Name	Aligned National Career Cluster Knowledge & Skills Statements	C	P	Alignment Comments	Task Comments
RANGE OF MOTION	**HLC01.01.01:** Use knowledge of human structure and function to conduct health care role.	3	3	The task requires knowledge of a variety of applications to human structure (e.g., explain body planes, directional terms).	The task provides an opportunity to integrate the math concepts associated with the diagnostic and therapeutic support of treatment and actions for increasing the physical health of a patient. Extensions enrich the activity into applied skill development using appropriate tools and resources.
	HLPB02.01: Utilize the processes to assess and report patient's/client's health status in order to monitor and document patient progress.	2	2	The task provides an opportunity to assess and report on the patient's health status and progress. Extensions would increase the level of alignment.	
	HLPA06.01: Evaluate patient/client needs, strengths, and problems in order to determine if treatment goals are being reached.	2	2	The task asks students to forecast if the patient will meet his treatment goals. The actual monitoring of the progress toward the treatment goal does not occur in the base task.	
	HLPB06.01: Interpret any given procedure, demonstrate knowledge of the purpose for each procedure, and perform the specific procedure to create diagnostic results.	2	2	The task allows for applying the understanding of tools and procedures to create diagnostic results. The extension requires actual use of goniometer to measure ROM and increases performance alignment.	