Middle School
Advanced One-Year
Model Course Program

Middle School Advanced Course

Achieve, Inc.

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Middle School Advanced Course
A Model One-Year Middle School Program

The content material described in this course contains the foundational understandings necessary to prepare students to pursue the study of algebra and geometry at a high school level, whether that content is encountered in traditionally organized or integrated courses. It is designed to build on a rigorous K–6 experience such as one indicated by the expectations outlined in Achieve's Elementary Mathematics Benchmarks, Grades K–6 or documents developed by the National Council of Teachers of Mathematics. It also builds on the National Assessment of Educational Progress (NAEP) elementary guidelines as well as the expectations set in many states' elementary grade standards. In particular, it is expected that students will come to this course with a strong conceptual foundation as well as computational facility with whole and rational numbers.

The mastery of this material in the single year-long middle school course described here can be accomplished only by very able and dedicated students. Students without the prerequisite knowledge outlined in Appendix A should instead be presented with this material in a two-year course sequence like that outlined in Achieve's model Middle School Course 1 and Middle School Course 2. During this Middle School Advanced Course, students will extend their understanding of the operations and properties of the rational number system to real numbers, including numbers expressed using exponents and roots. Important middle school concepts of ratio, rates, scaling, and similarity are studied. Algebraic expressions and relationships grow naturally out of work with numerical operations. Facility with variable expressions and numerical relationships expressed algebraically and graphically is critical to later success in more advanced mathematics courses. A major emphasis in this one-year course is on linear relationships; these are used to introduce students to the concept of a function and its multiple representations. Solving linear equations and connecting their solutions to the graph of the related linear function and to a contextual situation from which the equation might have arisen further prepare students for more rigorous mathematical modeling in later courses. This knowledge and skill also open the door to interesting and varied applications of the mathematics students are learning. Work with data analysis extends the algebraic lessons further into real life situations. Plane geometry as it relates to transformations and the geometry of circles as well as the in-depth study of slope will afford students opportunities to connect different branches of mathematics. Simple logical arguments that both verify and establish facts about geometric figures help solidify that knowledge. This Middle School Advanced Course also includes some basic work with probability. Two optional units are included that, while interesting and appropriate, may not be feasible within the time available in a typical 180-day school year. Upon completion of this course and its prerequisite materials, students should be prepared for success on eighth-grade state tests as well as the NAEP grade 8 assessment. They should also be prepared to successfully tackle algebra and geometry taught at the high school level.

Appropriate use of technology is expected in all work. In middle school this includes employing technological tools to assist students in creating graphs and data displays, transforming graphs, conceptualizing and analyzing geometric situations, and solving problems. Testing with and without technological tools is recommended.

How a particular subject is taught influences not only the depth and retention of the content of a course but also the development of skills in inquiry, problem solving, and critical thinking. Every opportunity should be taken to link the concepts of this middle school course to concepts students have encountered in earlier grades as well as to other disciplines. Students should be encouraged to be creative and innovative in their approach to problems, to be productive and persistent in seeking solutions, and to use multiple means to communicate their insights and understanding.

The **Major Concepts** below provide the focus for this one-year Middle School Advanced Course, which is intended to prepare students for a high school–level course in Algebra I or the first course in an Integrated Mathematics sequence. They should be taught using a variety of methods and applications so that students attain a deep understanding of these concepts. **Maintenance Concepts** should have been taught previously and are important foundational concepts that will be applied in this course. Continued facility with and understanding of the Maintenance Concepts is essential for success in the Major Concepts defined for this Middle School Advanced Course.

MAJOR CONCEPTS

- Real Numbers, Exponents, Roots, and the Pythagorean Theorem
- Variables and Expressions
- Functions
- Equations and Identities
- Geometric Reasoning, Representation, and Transformations
- Circles
- Ratios, Rates, Scaling, and Similarity
- Probability
- Question Formulation and Data Collection
- Number Bases

[OPTIONAL ENRICHMENT UNIT]

MAINTENANCE CONCEPTS

- Rational Number Operations
- Numeric Relationships
- Estimation and Approximation
- Integer Exponents
- Graphing in the Coordinate Plane
- Systems of Measurement
- Geometric Shapes and Measurement
- Perimeter, Area, Volume, and Surface Area
- Lines, Angles, and Triangles
- Elementary Data Analysis and Probability

A. Real Numbers, Exponents, Roots and the Pythagorean Theorem

Students extend the properties of computation with rational numbers to real number computation, categorize real numbers as either rational or irrational, and locate real numbers on the number line. Powers and roots are studied along with the Pythagorean theorem and its converse, a critical concept in its own right as well as a context in which numbers expressed using powers and roots arise. Students apply this knowledge to solve problems.

Successful students will:

A1 Use the definition of a *root* of a number to explain the relationship of powers and roots.

If $a^n = b$, for an integer $n \geq 0$, then a is said to be an nth root of b. When n is even and b > 0, we identify the unique a > 0 as the principal n^{th} root of b, written $\sqrt[n]{b}$.

a. Use and interpret the symbols $\sqrt{}$ *and* $\sqrt[3]{}$; informally explain why $\sqrt{ab} = \sqrt{a} \bullet \sqrt{b}$ and $\left(\sqrt{a}\right)^2 = a$, when *a > 0, b > 0,* $\sqrt{a^2} = |a|$, *and* $\sqrt[3]{a^3} = a$.

By convention, for a > 0, $\sqrt{a}$ *is used to represent the non-negative square root of a.*

b. Estimate square and cube roots and use calculators to find good approximations.

c. Make or refine an estimate for a square root using the fact that if $0 \leq a < n < b$, then $0 \leq \sqrt{a} < \sqrt{n} < \sqrt{b}$; make or refine an estimate for a cube root using the fact that if $a < n < b$, then $\sqrt[3]{a} < \sqrt[3]{n} < \sqrt[3]{b}$.

A2 Categorize real numbers as either rational or irrational and know that, by definition, these are the only two possibilities; extend the properties of computation with rational numbers to real number computation.

a. Approximately locate any real number on the number line.

b. Apply the definition of irrational number to identify examples and recognize approximations.

Square roots, cube roots, and n^{th} roots of whole numbers that are not respectively squares, cubes, and n^{th} powers of whole numbers provide the most common examples of irrational numbers. Pi (π) is another commonly cited irrational number.

c. Know that the decimal expansion of a rational number eventually repeats, perhaps ending in repeating zeros; use this to identify the decimal expansion of an irrational number as one that never ends and never repeats.

d. Recognize and use 22/7 and 3.14 as approximations for the irrational number represented by pi (π).

e. Determine whether the square, cube, and n^{th} roots of integers are integral or irrational when such roots are real numbers.

A3 Interpret and prove the Pythagorean theorem and its converse; apply the Pythagorean theorem and its converse to solve problems.

a. Determine distances between points in the Cartesian coordinate plane and relate the Pythagorean theorem to this process.

B. Variables and Expressions

In middle school, students work more with symbolic algebra than in the previous grades. Students develop an understanding of the different uses for variables, analyze mathematical situations and structures using algebraic expressions, determine if expressions are equivalent, and identify single-variable expressions as linear or non-linear.

Successful students will:

B1 Interpret and compare the different uses of variables and describe patterns, properties of numbers, formulas, and equations using variables.

While a variable has several distinct uses in mathematics, it is fundamentally just a number we either do not know yet or do not want to specify.

a. Compare the different uses of variables.

Examples: When $a + b = b + a$ is used to state the commutative property for addition, the variables a and b represent all real numbers; the variable a in the equation $3a - 7 = 8$ is a temporary placeholder for the one number, 5, that will make the equation true; the symbols C and r refer to specific attributes of a circle in the formula $C = 2\pi r$; the variable m in the slope-intercept form of the line, $y = mx + b$, serves as a parameter describing the slope of the line.

b. Express patterns, properties, formulas and equations using and defining variables appropriately for each case.

B2 Analyze and identify characteristics of algebraic expressions; evaluate, interpret, and construct simple algebraic expressions; identify and transform expressions into equivalent expressions; determine whether two algebraic expressions are equivalent.

Two algebraic expressions are equivalent if they yield the same result for every value of the variables in them. Great care must be taken to demonstrate that, in general, a finite number of instances is not sufficient to demonstrate equivalence.

a. Analyze expressions to identify when an expression is the sum of two or more simpler expressions (called terms) or the product of two or more simpler expressions (called factors). Analyze the structure of an algebraic expression and identify the resulting characteristics.

b. Identify single-variable expressions as linear or non-linear.

c. Evaluate a variety of algebraic expressions at specified values of their variables.

Algebraic expressions to be evaluated include polynomial and rational expressions as well as those involving radicals and absolute value.

d. Write linear and quadratic expressions representing quantities arising from geometric and real-world contexts.

e. Use commutative, associative, and distributive properties of number operations to transform simple expressions into equivalent forms in order to collect like terms or to reveal or emphasize a particular characteristic.

f. Rewrite linear expressions in the form $ax + b$ for constants a and b.

g. Choose different but equivalent expressions for the same quantity that are useful in different contexts.

Example: $p + 0.07p$ shows the breakdown of the cost of an item into the price p and the tax of 7%, whereas $(1.07)p$ is a useful equivalent form for calculating the total cost.

h. Demonstrate equivalence through algebraic transformations or show that expressions are not equivalent by evaluating them at the same value(s) to get different results.

i. Know that if each expression is set equal to *y* and the graph of all ordered pairs that satisfy one of these new equations is identical to the graph of all ordered pairs that satisfy the other, then the expressions are equivalent.

C. Functions

Middle school students increase their experience with functional relationships and begin to express and understand them in more formal ways. They distinguish between relations and functions and convert flexibly among the various representations of tables, symbolic rules, verbal descriptions, and graphs. A major focus at this level is on linear functions, recognizing linear situations in context, describing aspects of linear functions such as slope as a constant rate of change, identifying x- and y-intercepts, and relating slope and intercepts to the original context of the problem.

Successful students will:

C1 Determine whether a relationship is or is not a function; represent and interpret functions using graphs, tables, words, and symbols

In general, a function is a rule that assigns a single element of one set—the output set—to each element of another set—the input set. The set of all possible inputs is called the domain of the function, while the set of all outputs is called the range.

a. Identify the independent (input) and dependent (output) quantities/variables of a function.

b. Make tables of inputs x and outputs $f(x)$ for a variety of rules that take numbers as inputs and produce numbers as outputs.

c. Define functions algebraically, *e.g.* $g(x) = 3 + 2(x - x^2)$.

d. Create the graph of a function f by plotting and connecting a sufficient number of ordered pairs $(x, f(x))$ in the coordinate plane.

e. Analyze and describe the behavior of a variety of simple functions using tables, graphs, and algebraic expressions.

f. Construct and interpret functions that describe simple problem situations using expressions, graphs, tables, and verbal descriptions and move flexibly among these multiple representations.

C2 Analyze and identify linear functions of one variable; know the definitions of *x*- and *y*-intercepts and slope, know how to find them and use them to solve problems.

A function exhibiting a rate of change (slope) that is constant is called a linear function. A constant rate of change means that for any pair of inputs x_1 and x_2, the ratio of the corresponding change in value $f(x_2) - f(x_1)$ to the change in input $x_2 - x_1$ is constant (i.e., it does not depend on the inputs).

a. Explain why any function defined by a linear algebraic expression has a constant rate of change.

b. Explain why the graph of a linear function defined for all real numbers is a straight line, identify its constant rate of change, and create the graph.

c. Determine whether the rate of change of a specific function is constant; use this to distinguish between linear and nonlinear functions.

d. Know that a line with slope equal to zero is horizontal and represents a function while the slope of a vertical line is undefined and cannot represent a function.

C3 Express a linear function in several different forms for different purposes.

a. Recognize that in the form $f(x) = mx + b$, m is the slope, or constant rate of change of the graph of f, that b is the y-intercept and that in many applications of linear functions, b defines the initial state of a situation; express a function in this form when this information is given or needed.

b. Recognize that in the form $f(x) = m(x - x_0) + y_0$, the graph of $f(x)$ passes through the point (x_0, y_0); express a function in this form when this information is given or needed.

C4 Recognize contexts in which linear models are appropriate; determine and interpret linear models that describe linear phenomena; express a linear situation in terms of a linear function $f(x) = mx + b$ and interpret the slope (m) and the y-intercept (b) in terms of the original linear context.

Common examples of linear phenomena include distance traveled over time for objects traveling at constant speed; shipping costs under constant incremental cost per pound; conversion of measurement units (e.g., pounds to kilograms or degrees Celsius to degrees Fahrenheit); cost of gas in relation to gallons used; the height and weight of a stack of identical chairs.

C5 Recognize, graph, and use direct proportional relationships.

A linear function in which $f(0) = 0$ represents a direct proportional relationship. The linear function $f(x) = kx$, where k is constant describes a direct proportional relationship.

a. Show that the graph of a direct proportional relationship is a line that passes through the origin (0, 0) whose slope is the constant of proportionality.

b. Compare and contrast the graphs of $x = k$, $y = k$, and $y = kx$, where k is a constant.

D. Equations and Identities

In this middle school course, students begin the formal study of equations. They solve linear equations and solve and graph linear inequalities in one variable. They graph equations in two variables, relating features of the graphs to the related single-variable equations. Solving systems of two linear equations in two variables graphically and understanding what it means to be a solution of such a system is also included in this unit. Interwoven with the development of these skills, students use linear equations, inequalities, and systems of linear equations to solve problems in context and interpret the solutions and graphical representations in terms of the original problem.

Successful students will:

D1 Distinguish among an equation, an expression, and a function; interpret identities as a special type of equation and identify their key characteristics.

An identity is an equation for which all values of the variables are solutions. Although an identity is a special type of equation, there is a difference in practice between the methods for solving equations that have a small number of solutions and methods for proving identities. For example, $(x+2)^2 = x^2 + 4x + 4$ *is an identity which can be proved by using the distributive property, whereas* $(x+2)^2 = x^2 + 3x + 4$ *is an equation that can be solved by collecting all terms on one side.*

a. Know that solving an equation means finding all its solutions and predict the number of solutions that should be expected for various simple equations and identities.

b. Explain why solutions to the equation $f(x) = g(x)$ are the x-values (abscissas) of the set of points in the intersection of the graphs of the functions $f(x)$ and $g(x)$.

c. Recognize that $f(x) = 0$ is a special case of the equation $f(x) = g(x)$ and solve the equation $f(x) = 0$ by finding all values of x for which $f(x) = 0$.

The solutions to the equation $f(x) = 0$ *are called roots of the equation or zeros of the function. They are the values of* x *where the graph of the function* f *crosses the* x*-axis. In the special case where* $f(x)$ *equals 0 for all values of* x*,* $f(x) = 0$ *represents a constant function where all elements of the domain are zeros of the function.*

d. Use identities to transform expressions.

D2 Solve linear equations and solve and graph the solution of linear inequalities in one variable.

Common problems are those that involve break-even time, time/rate/distance, percentage increase or decrease, ratio and proportion.

a. Solve equations using the facts that equals added to equals are equal and that equals multiplied by equals are equal. More formally, if $A = B$ and $C = D$, then $A + C = B + D$ and $AC = BD$. Use the fact that a linear expression $ax + b$ is formed using the operations of multiplication by a constant followed by addition to solve an equation $ax + b = 0$ by reversing these steps.

Be alert to anomalies caused by dividing by 0 (which is undefined), or by multiplying both sides by 0 (which will produce equality even when things were originally unequal).

b. Graph a linear inequality in one variable and explain why the graph is always a half-line (open or closed). Know that the solution set of a linear inequality in one variable is infinite, and contrast this with the solution set of a linear equation in one variable.

c. Explain why, when both sides of an inequality are multiplied or divided by a negative number, the direction of the inequality is reversed but that when all other basic operations involving non-zero numbers are applied to both sides, the direction of the inequality is preserved.

D3 Recognize, represent, and solve problems that can be modeled using linear equations in two variables and interpret the solution(s) in terms of the context of the problem.

a. Rewrite a linear equation in two variables in any of three forms: $ax + by = c$, $ax + by + c = 0$, or $y = mx + b$; select a form depending upon how the equation is to be used.

b. Know that the graph of a linear equation in two variables consists of all points (x, y) in the coordinate plane that satisfy the equation and explain why, when x can be any real number, such graphs are straight lines.

c. Identify the relationship between linear functions in one variable, x maps to $f(x)$ and linear equations in two variables $f(x) = y$ or $f(x) - y = 0$; explain why the solution to an equation in standard (or polynomial) form ($ax + b = 0$) will be the point where the graph of $f(x) = ax + b$ crosses the x-axis.

d. Identify the solution of an equation that is in the form $f(x) = g(x)$ and relate the solution to the x-value (*abscissa*) of the point at which the graphs of the functions $f(x)$ and $g(x)$ intersect.

e. Know that pairs of non-vertical lines have the same slope if and only if they are parallel (or the same line) and slopes that are negative reciprocals if and only if they are perpendicular; apply these relationships to analyze and represent equations.

f. Represent linear relationships using tables, graphs, verbal statements, and symbolic forms; translate among these forms to extract information about the relationship.

D4 Determine the solution to application problems modeled by two linear equations and interpret the solution set in terms of the situation.

a. Determine either through graphical methods or comparing slopes whether a system of two linear equations has one solution, no solutions, or infinitely many solutions, and know that these are the only possibilities.

b. Represent the graphs of two linear equations as two intersecting lines when there is one solution, parallel lines when there is no solution, and the same line when there are infinitely many solutions.

c. Use the graph of two linear equations in two variables to suggest solution(s).

Since the solution is a set of ordered pairs that satisfy the equations, it follows that these ordered pairs must lie on the graph of each of the equations in the system; the point(s) of intersection of the graphs is (are) the solution(s) to the system of equations.

d. Recognize and solve problems that can be modeled using two linear equations in two variables.

Examples: Break-even problems, such as those comparing costs of two services.

E. Geometric Representation and Transformations

Coordinate geometry affords middle school students the opportunity to make valuable connections between algebra concepts and geometry representations such as slope and distance. Students extend their elementary school experiences with transformations as specific motions in two-dimensions to transformations of figures in the coordinate plane. They describe the characteristics of transformations that preserve distance, relating them to congruence.

Successful students will:

E1 Represent and explain the effect of translations, rotations, and reflections of objects in the coordinate plane.

a. Identify certain transformations (translations, rotations, and reflections) of objects in the plane as *rigid* motions and describe their characteristics; know that they preserve distance in the plane.

b. Demonstrate the meaning and results of the translation, rotation, and reflection of an object through drawings and experiments.

c. Identify corresponding sides and angles between objects and their images after a rigid transformation.

d. Show how any rigid motion of a figure in the plane can be accomplished through a sequence of translations, rotations, and reflections.

E2 Represent and interpret points, lines, and two-dimensional geometric objects in a coordinate plane; calculate the slope of a line in a coordinate plane.

a. Determine the area of polygons in the coordinate plane.

b. Know how the word *slope* is used in common non-mathematical contexts, give physical examples of slope, and calculate slope for given examples.

c. Find the slopes of physical objects (roads, roofs, ramps, stairs) and express the answers as a decimal, ratio, or percent.

d. Interpret and describe the slope of parallel and perpendicular lines in a coordinate plane.

e. Show that the calculated slope of a line in a coordinate plane is the same no matter which two distinct points on the line one uses to calculate the slope.

g. Use coordinate geometry to determine the perpendicular bisector of a line segment.

F. Circles

The main focus of this unit is on the study of circles, the relationships among its parts, the development of the formulas for the area and circumference and methods for approximating π.

Successful students will:

F1 Identify and explain the relationships among the radius, diameter, circumference, and area of a circle; know and apply formulas for the circumference and area of a circle, semicircle, and quarter-circle.

a. Identify the relationship between the circumference of a circle and its radius or diameter as a direct proportion and between the area of a circle and the square of its radius or the square of its diameter as a direct proportion.

b. Demonstrate why the formula for the area of a circle (radius times one-half of its circumference) is plausible and makes geometric sense.

c. Show that for any circle, the ratio of the circumference to the diameter is the same as the ratio of the area to the square of the radius and that these ratios are the same for different circles; identify the constant ratio $A/r^2 = \frac{1}{2}Cr/r^2 = C/2r = C/d$ as the number π and know that although the rational numbers 3.14, or $\frac{22}{7} = 3\frac{1}{7}$ are often used to approximate π, they are not the actual values of the irrational number π.

d. Identify and describe methods for approximating π.

G. Ratios, Rates, Scaling, and Similarity

In conjunction with the study of rational numbers, middle school students examine ratios, rates and proportionality both procedurally and conceptually. Proportionality concepts connect many areas of the curriculum, number, similarity, scaling, slope and probability and serve as a foundation for future mathematics study. Examining proportionality first with numbers then geometrically with similarity concepts and scaling begins to establish important understandings for more formal study of these concepts in high school algebra and geometry courses.

Successful students will:

G1 Use ratios, rates, and derived quantities to solve problems.

a. Interpret and apply measures of change such as percent change and rates of growth.

b. Calculate with quantities that are derived as ratios and products.

Examples: Interpret and apply ratio quantities including velocity and population density using units such as feet per second and people per square mile; interpret and apply product quantities including area, volume, energy, and work using units such as square meters, kilowatt hours, and person days.

c. Solve data problems using ratios, rates and product quantities.

d. Create and interpret scale drawings as a tool for solving problems.

A scale drawing is a representation of a figure that multiplies all the distances between corresponding points by a fixed positive number called the scale factor

G2 Analyze and represent the effects of multiplying the linear dimensions of an object in the plane or in space by a constant scale factor, *r*.

a. Use ratios and proportional reasoning to apply a scale factor to a geometric object, a drawing, or a model, and analyze the effect.

b. Describe the effect of a scale factor r on length, area, and volume.

G3 Interpret the definition and characteristics of similarity for figures in the plane and apply to problem solving situations.

Informally, two geometric figures in the plane are similar if they have the same shape. More formally, having the same shape means that one figure can be transformed onto the other by applying a scale factor.

a. Apply similarity in practical situations; calculate the measures of corresponding parts of similar figures.

b. Use the concepts of similarity to create and interpret scale drawings.

H. Probability

Students have an opportunity in this unit to apply both their rational number and proportional reasoning skills to probability situations. Students use theoretical probability and proportions to predict outcomes of simple events. Frequency distributions are examined and created to analyze the likelihood of events. The Law of Large Numbers is used to link experimental and theoretical probabilities.

Successful students will:

H1 Describe the relationship between probability and relative frequency; use a probability distribution to assess the likelihood of the occurrence of an event.

a. Recognize and use relative frequency as an estimate for probability.

If an action is repeated n times and a certain event occurs b times, the ratio b/n is called the relative frequency of the event occurring.

b. Use theoretical probability, where possible, to determine the most likely result if an experiment is repeated a large number of times.

c. Identify, create, and describe the key characteristics of frequency distributions of discrete and continuous data.

A frequency distribution shows the number of observations falling into each of several ranges of values; if the percentage of observations is shown, the distribution is called a relative frequency distribution. Both frequency and relative frequency distributions are portrayed through tables, histograms, or broken-line graphs.

d. Analyze and interpret actual data to estimate probabilities and predict outcomes.

Example: In a sample of 100 randomly selected students, 37 of them could identify the difference in two brands of soft drink. Based on these data, what is the best estimate of how many of the 2,352 students in the school could distinguish between the soft drink?

e. Compare theoretical probabilities with the results of simple experiments (e.g., tossing number cubes, flipping coins, spinning spinners).

f. Explain how the Law of Large Numbers explains the relationship between experimental and theoretical probabilities.

The Law of Large Numbers indicates that if an event of probability p is observed repeatedly during independent repetitions, the ratio of the observed frequency of that event to the total number of repetitions approaches p as the number of repetitions becomes arbitrarily large.

g. Use simulations to estimate probabilities.

h. Compute and graph cumulative frequencies.

I. Question Formulation and Data Collection

Students learn to design a study to answer a question; collect, organize, and summarize data; communicate the results; and make decisions about the findings. Technology is utilized both to analyze and display data. Students expand their repertoire of graphs and statistical measures and begin the use of random sampling in sample surveys. They assess the role of random assignment in experiments. They look critically at data studies and reports for possible sources of bias or misrepresentation. Students are able to use their knowledge of slope to analyze lines of best fit in scatter plots and make predictions from the data further connecting their algebra and data knowledge.

Successful students will:

I1 Formulate questions about a phenomenon of interest that can be answered with data; design a plan to collect appropriate data; collect and record data; display data using tables, charts, or graphs; evaluate the accuracy of the data.

a. Recognize the need for data; understand that data are numbers in context (with units) and identify units. Define measurements that are relevant to the questions posed; organize written or computerized data records, making use of computerized spreadsheets.

b. Understand the differing roles of a census, a sample survey, an experiment, and an observational study.

c. Select a design appropriate to the questions posed.

d. Use random sampling in sample surveys and random assignment in experiments, introducing random sampling as a "fair" way to select an unbiased sample.

I2 Represent both univariate and bivariate quantitative (measurement) data accurately and effectively.

a. Represent univariate data; make use of line plots (dot plots), stem-and-leaf plots, and histograms.

b. Represent bivariate data; make use of scatter plots.

c. Describe the shape, center, and spread of data distributions.

Example: A scatter plot used to represent bivariate data may have a linear shape; a trend line may pass through the mean of the x and y variables; its spread is shown by the vertical distances between the actual data points and the line.

d. Identify and explain misleading uses of data by considering the completeness and source of the data, the design of the study, and the way the data are analyzed and displayed.

Examples: Determine whether the height or area of a bar graph is being used to represent the data; evaluate whether the scales of a graph are consistent and appropriate or whether they are being adjusted to alter the visual information conveyed.

I3 Summarize, compare, and interpret data sets by using a variety of statistics. Use percentages and proportions (relative frequencies) to summarize univariate categorical data.

a. Use conditional (row or column) percentages and proportions to summarize bivariate categorical data.

b. Use measures of center (mean and median) and measures of spread (percentiles, quartiles, and interquartile range) to summarize univariate quantitative data.

c. Use trend lines (linear approximations or best-fit line) to summarize bivariate quantitative data.

d. Graphically represent measures of center and spread (variability) for quantitative data.

e. Interpret the slope of a linear trend line in terms of the data being studied.

f. Use box plots to compare key features of quantitative data distributions.

I4 Read, interpret, interpolate, and judiciously extrapolate from graphs and tables and communicate the results.

a. State conclusions in terms of the question(s) being investigated.

b. Use appropriate statistical language when reporting on plausible answers that go beyond the data actually observed.

c. Use oral, written, graphic, pictorial and multi-media methods to create and present manuals and reports.

I5 Determine whether a scatter plot suggests a linear trend.

a. Visually determine a line of good fit to estimate the relationship in bivariate data that suggests a linear trend.

b. Identify criteria that might be used to assess how good the fit is.

The following unit is an extension or enrichment unit, which while interesting and appropriate, may not be feasible time-wise in a traditional 180-day school year.

J. Number Bases [OPTIONAL ENRICHMENT UNIT]

This should be used as an optional unit of study if time permits. Using their understanding of the base-10 number system, students represent numbers in other bases. Computers and computer graphics have made much more important the knowledge of how to work with different base systems, particularly binary.

Successful students will:

J1 Identify key characteristics of the base-10 number system and adapt them to the binary number base system.

a. Represent and interpret numbers in the binary number system.

b. Apply the concept of base-10 place value to understand representation of numbers in other bases.

Example: In the base-8 number system, the 5 in the number 57,273 represents 5 x 8^4.

c. Convert binary to decimal and vice versa.

d. Encode data and record measurements of information capacity using the binary number base system.

Appendix A

The following expectations, which are included in the model two-year middle school course sequence (Middle Course 1 and Middle School Course 2), are essential prerequisites for success in the one-year middle school program (Middle School Advanced Course). Students must develop proficiency in these expectations prior to embarking upon this one-year advanced course. This means that schools opting for this one-year Middle School Advanced Course must adjust the mathematics curriculum in earlier grades to include these expectations.

(MSAPK=MIDDLE SCHOOL ADVANCED PRIOR KNOWLEDGE)

PK.A. Number Representation and Computation

Successful students will:

PK.A1 Extend and apply understanding about rational numbers; translate among different representations of rational numbers.

Rational numbers are those that can be expressed in the form $\frac{p}{q}$ *where p and q are integers and q ≠ 0.*

a. Use inequalities to compare rational numbers and locate them on the number line; apply basic rules of inequalities to transform expressions involving rational numbers.

PK.A2 Apply the properties of computation (e.g., commutative property, associative property, distributive property) to positive and negative rational number computation; know and apply effective methods of calculation with rational numbers.

a. Demonstrate understanding of the algorithms for addition, subtraction, multiplication, and division (non-zero divisor) of numbers expressed as fractions, terminating decimals, or repeating decimals by applying the algorithms and explaining why they work

b. Add, subtract, multiply, and divide (non-zero divisor) rational numbers and explain why these operations always produce another rational number.

c. Interpret parentheses and employ conventional order of operations in a numerical expression, recognizing that conventions are universally agreed upon rules for operating on expressions.

d. Check answers by estimation or by independent calculations, with or without calculators and computers.

e. Solve practical problems involving rational numbers.

   Examples: Calculate markups, discounts, taxes, tips, average speed.

PK.A3 Recognize, describe, extend, and create well-defined numerical patterns.

A pattern is a sequence of numbers or objects constructed using a simple rule. Of special interest are arithmetic sequences, those generated by repeated addition of a fixed number, and geometric sequences, those generated by repeated multiplication by a fixed number.

PK.A4 Know and apply the Fundamental Theorem of Arithmetic.

Every positive integer is either prime itself or can be written as a unique product of primes (ignoring order).

a. Identify prime numbers; describe the difference between prime and composite numbers; determine divisibility rules (2, 3, 5, 9, 10), explain why they work, and use them to help factor composite numbers

b. Determine the greatest common divisor and least common multiple of two whole numbers from their prime factorizations; explain the meaning of the greatest common divisor (greatest common factor) and the least common multiple and use them in operations with fractions.

c. Use greatest common divisors to reduce fractions $\frac{n}{m}$ and ratios $n{:}m$ to an equivalent form in which the gcd $(n, m) = 1$.

Fractions n/m in which gcd (n, m) = 1 are said to be in lowest terms.

d. Write equivalent fractions by multiplying both numerator and denominator by the same non-zero whole number or dividing by common factors in the numerator and denominator.

e. Add and subtract fractions by using the least common multiple (or any common multiple) of denominators.

PK.A5 Identify situations where estimates are appropriate and use estimates to predict results and verify the reasonableness of calculated answers.

a. Use rounding, regrouping, percentages, proportionality, and ratios as tools for mental estimation.

b. Develop, apply, and explain different estimation strategies for a variety of common arithmetic problems.

Examples: Estimating tips, adding columns of figures, estimating interest payments, estimating magnitude.

c. Explain the phenomenon of rounding error, identify examples, and, where possible, compensate for inaccuracies it introduces.

Examples: Analyzing apportionment in the U.S. House of Representatives; creating data tables that sum properly; analyzing what happens to the sum if you always round down when summing 100 terms.

PK.A6 Use the rules of exponents and roots to simplify and evaluate expressions.

a. Evaluate expressions involving positive integer exponents and interpret such exponents in terms of repeated multiplication.

PK.A7 Know and apply the definition of absolute value.

The absolute value is defined by $|a| = a$ if $a > 0$ and $|a| = -a$ if $a < 0$.

a. Interpret absolute value as distance from zero.

b. Interpret absolute value of a difference as "distance between” on the number line.

PK.A8 Analyze and apply simple algorithms

a. Identify and give examples of simple algorithms.

An algorithm is a procedure (a finite set of well-defined instructions) for accomplishing some task that, given an initial state, will terminate in a well-defined end-state. Recipes and assembly instructions are everyday examples of algorithms.

b. Analyze and compare simple computational algorithms.

Examples: Write the prime factorization for a large composite number; determine the least common multiple for two positive integers; identify and compare mental strategies for computing the total cost of several objects.

c. Analyze and apply the iterative steps in standard base-10 algorithms for addition and multiplication of numbers.

PK.B. Measurement Systems

Successful students will:

PK.B1 Make, record, and interpret measurements.

a. Recognize that measurements of physical quantities must include the unit of measurement, that most measurements permit a variety of appropriate units, and that the numerical value of a measurement depends on the choice of unit; apply these ideas when making such measurements.

b. Recognize that real-world measurements are approximations; identify appropriate instruments and units for a given measurement situation, taking into account the precision of the measurement desired.

c. Plan and carry out both direct and indirect measurements.

Indirect measurements are those that are calculated based on actual recorded measurements.

d. Apply units of measure in expressions, equations, and problem situations; when necessary, convert measurements from one unit to another within the same system.

e. Use measures of weight, money, time, information, and temperature; identify the name and definition of common units for each kind of measurement.

f. Record measurements to reasonable degrees of precision, using fractions and decimals as appropriate.

A measurement context often defines a reasonable level of precision to which the result should be reported.

Example: The U.S. Census bureau reported a national population of 299,894,924 on its Population Clock in mid-October of 2006. Saying that the U.S. population is 3 hundred million ($3x10^8$) is accurate to the nearest million and exhibits one-digit precision. Although by the end of that month the population had surpassed 3 hundred million, $3x10^8$ remained accurate to one-digit precision.

PK.B2 Identify and distinguish among measures of length, area, surface area, and volume. Calculate perimeter, area, surface area, and volume.

a. Calculate the perimeter and area of triangles, quadrilaterals, and shapes that can be decomposed into triangles and quadrilaterals that do not overlap; know and apply formulas for the area and perimeter of triangles and rectangles to derive similar formulas for parallelograms, rhombi, trapezoids, and kites.

b. Given the slant height, determine the surface area of right prisms and pyramids whose base(s) and sides are composed of rectangles and triangles; know and apply formulas for the surface area of right circular cylinders, right circular cones, and spheres; explain why the surface area of a right circular cylinder is a rectangle whose length is the circumference of the base of the cylinder and whose width is the height of the cylinder.

c. Given the slant height, determine the volume of right prisms, right pyramids, right circular cylinders, right circular cones, and spheres.

d. Estimate lengths, areas, surface areas, and volumes of irregular figures and objects.

PK.C. Angles and Triangles

Successful students will:

PK.C1 Know the definitions and properties of angles and triangles in the plane and use them to solve problems.

a. Know and apply the definitions and properties of complementary, supplementary, interior, and exterior angles.

b. Know and distinguish among the definitions and properties of vertical, adjacent, corresponding, and alternate interior angles; identify pairs of congruent angles and explain why they are congruent.

PK.C2 Know and verify basic theorems about angles and triangles.

a. Know the triangle inequality and verify it through measurement.

In words, the triangle inequality states that any side of a triangle is shorter than the sum of the other two sides; it can also be stated clearly in symbols: If a, b, and c are the lengths of three sides of a triangle, then $a < b + c$, $b < a + c$, *and* $c < a + b$.

b. Verify that the sum of the measures of the interior angles of a triangle is 180°.

c. Verify that each exterior angle of a triangle is equal to the sum of the opposite interior angles.

d. Show that the sum of the interior angles of an n-sided convex polygon is $(n - 2) \times 180°$.

e. Explain why the sum of exterior angles of a convex polygon is 360°.

PK.D. 3-Dimensional Geometry

Successful students will:

PK.D1 Visualize solids and surfaces in three-dimensional space.

a. Relate a net, top-view, or side-view to a three-dimensional object that it might represent. Visualize and be able to reproduce solids and surfaces in three-dimensional space when given two-dimensional representations (e.g., nets, multiple views).

b. Interpret the relative position and size of objects shown in a perspective drawing.

c. Visualize, describe and identify three-dimensional shapes in different orientations; draw two-dimensional representations of three-dimensional object by hand and using software; sketch two-dimensional representations of basic three-dimensional objects such as cubes, spheres, pyramids, and cones.

d. Create a net, top-view, or side-view of a three-dimensional object by hand or using software; visualize, describe, or sketch the cross-section of a solid cut by a plane that is parallel or perpendicular to a side or axis of symmetry of the solid.

PK.E. Data Analysis

Successful students will:

PK.E1 Represent both univariate and bivariate categorical data accurately and effectively.

a. For univariate data, make use of frequency and relative frequency tables and bar graphs; for bivariate data make use of two-way frequency and relative frequency tables and bar graphs.

PK.F. Probability

Successful students will:

PK.F1 Represent probabilities using ratios and percents; use sample spaces to determine the (theoretical) probabilities of events; compare probabilities of two or more events and recognize when certain events are equally likely.

a. Calculate theoretical probabilities in simple models (e.g., number cubes, coins, spinners).

b. Know and use the relationship between probability and odds.

The odds of an event occurring is the ratio of the number of favorable outcomes to the number of unfavorable outcomes, whereas the probability is the ratio of favorable outcomes to the total number of possible outcomes.